Higher rank graphs: geometry, symmetry, dynamics

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Abstracts

Ara, Pere

The groupoids of adaptable separated graphs and their type semigroups (II)

Using the technology introduced in Enrique Pardo's talk, we will show that the type semigroups of groupoids obtained from adaptable separated graphs in this way include all finitely generated conical refinement monoids. Moreover, we study the Steinberg algebra of these groupoids, and we construct suitable universal localization of them. These universal localization turns out to be von Neumann regular rings, whose V-monoids are isomorphic to the given type semigroups. So, we solve the Representation Problem for von Neumann regular rings in the case of finitely generated monoids.

Chouraqui, Fabienne

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Let $\mathrm{R}: \mathrm{V} \otimes V \to V \otimes V$ be a linear operator, where V is a vector space. The QYBE is the equality $\mathrm{R}^{12} R^{13} R^{23} = R^{23} R^{13} R^{12}$ of linear transformations on $\mathrm{V} \otimes V \otimes V$, where R^{ij} means R acting on the ith and j th components. A set-theoretical solution of this equation is a solution for which V is a vector space spanned by a set X and R is the linear operator induced by a mapping $\mathrm{X} \times X \to X \times X$. The study of these was suggested by Drinfeld. To each such solution is associated a group called the structure group. The structure groups of set-theoretical solutions (X,S) of the QYBE that are non-degenerate, involutive and braided have been extensively studied. Garside groups were first introduced by P.Dehornoy and L.Paris in 1990. In many ways, Garside groups extend braid groups and more generally finite-type Artin groups. These are torsion-free groups with a word and conjugacy problems solvable, and they are groups of fractions of monoids with a structure of lattice with respect to left and right divisibilities. We show the structure group of a non-degenerate, involutive and braided set-theoretical solution (X,S) of the QYBE is a Garside group that satisfies many interesting properties. Conversely, given a Garside group G with a particular presentation, there exists a non-degenerate, involutive and braided set-theoretical solution such that G is its structure group. In this talk, I will introduce the Garside groups in general, the set-theoretical solutions of the QYBE, and describe the connection between these theories arising from different domains of research.

Clark, Lisa O.

Ample groupoid algebras

A groupoid is a generalisation of a group where the binary operation is only partially defined. In this talk, I will introduce ample groupoids, which are groupoids that come with a particularly nice topological structure. Then, I will describe the Steinberg algebra and groupoid C*-algebras of an ample groupoid. These algebras generalise the (discrete) group algebra and group C*-algebras respectively. We will also look at some classes of examples that come from directed and higher-rank graphs.

Evans, Gwion
On the K-theory of k-graph C-algebras (2001: a spectral odyssey)*
K-theory is a powerful tool in the theory and applications of C*-algebras. It is at the heart of Elliott's classification programme for amenable C*-algebras, provides useful topological invariants of certain dynamical systems, bridges seemingly disparate areas of mathematics and reveals deep insights in mathematical physics. The core data consists of two abelian groups associated to a C*-algebra, the so-called K-groups, the identification of which can be highly non-trivial. Procedures to identify K-groups for families of C*-algebras are therefore highly sought after. An account of the first attempts to compute K-groups for families of higher-rank graph C*-algebras will be given along with a survey of the current state of play of the K-theory of higher-rank graph C*-algebras and a discussion of its applications. Prior knowledge of the K-theory of C*-algebras will not be required as explanations of all relevant concepts will be given.

Farsi, Carla
Representations for k-graph C-algebras*
This talk will focus on representation theory for k-graph C*-algebras. After introducing the main concepts and definitions, we develop some of the relevant theory. In particular we will look in detail at the case of monic representations of k-graph C*-algebras, in both the finite (joint work with Gillaspy, Jorgensen, Kang and Packer), and also in the more general row-finite case (joint work with Gillaspy and Goncalves).

Garrido Angulo, Alejandra
Locally compact piecewise full groups of Cantor homeomorphisms
Piecewise full groups (a.k.a topological full groups) of homeomorphisms of the Cantor set are a general construction obtained from actions on the Cantor set (more generally, from groupoids with unit space the Cantor set) that were initially introduced as invariants of minimal Cantor dynamical systems and have been used to produce examples of finitely generated simple groups with extra properties, e.g., finite presentability and amenability (Juschenko-Monod), torsion and intermediate growth (Nekrashevych). A non-discrete locally compact example of a piecewise full group is the much-studied group of aplmost automorphisms of a regular tree (Neretin's group), which is abstractly simple, and also compactly generated (therefore Polish). I will report on joint work with C. Reid and D. Robertson where we investigate the possible Polish topologies that piecewise full groups can admit and show that there is at most one, determined by the group structure. We also give conditions on the 'input' for the piecewise full group to be compactly generated. Thus we obtain many new 'relatives' of Neretin's group.

Kumjian, Alex
Homology and cohomology of higher rank graphs
We introduce the cubical homology and cohomology of a higher rank graph and discuss some fundamental theorems. We also discuss the categorical cohomology and observe that the two cohomology theories coincide by a recent result of Gillaspy and Wu. If there is time we construct the cubical set associated to a higher rank graph.

Lawson, Mark
Non-commutative Stone duality: groups, groupoids and pseudogroups
This talk will be in two parts. In the first part, I will outline the theory of non-commutative Stone duality concentrating on the monoid/compact and the Boolean cases. In the second part, I will focus on the implications of this theory for 1-vertex, higher-rank graphs. This is joint work with Daniel Lenz and Alina Vdovina.

Nucinkis, Brita
An introduction to Thompson's groups and their generalisations
In this talk I will give an introduction to several ways of defining Thompson's groups F and V and their generalisations, including the higher dimensional Thompson groups of Brin. I will also give an overview about some of their most interesting properties such as finiteness conditions and simplicity. If time permits, I will also introduce an example of a Thompson group with irrational slopes, which has some surprising features. This part it joint work with Burillo and Reeves.

Pardo, Enrique

The groupoids of adaptable separated graphs and their type semigroups (I)

Given an adaptable separated graph, we construct an associated groupoid and develop the technology to explore its type semigroup. Specifically, we first attach to each adaptable separated graph a corresponding inverse semigroup, and we show that its tight groupoid is an ample, Hausdorff, amenable groupoid.

Pask, David

Higher rank graphs and higher dimensional shift spaces

Higher dimensional shift spaces are notoriously difficult to study due to decidability problems. No general theory has been developed. As a result several sub-classes have been analysed, such as the dynamical systems of algebraic origin due to Lind and Schmidt. In this talk we will examine higher dimensional shift spaces associated to higher rank graphs. In one dimension everything works well. The shift space associated to a 1-graph is a shift of finite type and up to conjugacy) every shift of finite type occurs this way. However, as we shall discover, in higher dimensions things are more complicated/interesting.

Whittaker, Mike

An introduction to graph algebras

In this expository talk I will introduce graph algebras and their properties. Graph algebras are generalisations of Cuntz-Krieger algebras, and their C*-algebras model the combinatorics of the path space of the underlying directed graph. In fact, C*-algebraic properties of a graph algebras can typically be seen at the level of the graph itself. This feature gives us insight into similar properties for more complicated C*-algebras. I plan to discuss some of the fundamental features of graph algebras along with some more recent developments. No prior knowledge of graph algebras will be assumed in this talk.

Sims, Aidan

An introduction to higher-rank graphs

Kumjian and Pask introduced the notion of a higher-rank graph in 2000 as a combinatorial model for the higher-rank Cuntz-Krieger algebras discovered by Robertson and Steger in their work on actions of groups on buildings. Roughly speaking, a higher-rank graph is like the path-category of a directed graph, except that paths have a k-dimensional "degree" in place of a 1-dimensional length. In this talk I will outline what a k-graph is and how to describe k-graphs (and construct examples) using k-coloured directed graphs, and describe some of the standard constructions appearing in the literature. I will mention C*-algebras where they are relevant along the way, but will focus mainly on a no-assumed-knowledge introduction to k-graphs themselves.

Spielberg, Jack

Some generalizations of higher rank graphs

The C*-algebra of a directed graph was first defined by generators and relations in analogy with Cuntz-Krieger algebras. For higher rank graphs the situation was similar, although the groupoid description was born simultaneously in the original paper of Kumjian and Pask. However, the rigidity of the axioms for higher rank graphs makes their study much more difficult than that of directed graphs. For example, the algebra of a directed graph can be realized as the inductive limit of (relative Toeplitz) graph algebras over any family of finite subgraphs having union the original graph. A k-graph, on the other hand, need not have any finite sub k-graphs. It turns out that only the most basic properties of a higher rank graph, namely, composition and units, are needed to define its C*-algebra, and this process applies much more generally. This provides some partial remedy for the difficulties mentioned. I will describe this construction and how it may be of use in the study of higher rank graphs.

Steinberg, Benjamin
Higher rank graphs, k-subshifts and k-automata
Given a k-graph Λ we construct a Markov space M, and a collection of k pairwise commuting cellular automata on M, providing for a factorization of the shift. Iterating these maps we obtain an action of $\mathbf{N}^k$ on M which is then used to form a semidirect product groupoid. M rtimes $\mathbf{N}^k$. This groupoid turns out to be identical to the path groupoid constructed by Kumjian and Pask, and hence its C*-algebra is isomorphic to the higher rank graph C*-algebra of Λ. This is joint work with Ruy Exel.

Valette, Alain
Explicit Baum-Connes for lamplighter groups over finite groups
After a brief general introduction to the Baum-Connes conjecture, we focus on lamplighter groups $\mathrm{G}=\mathrm{F}\wr\mathbb{Z}$, with Fa finite group. We compute the K-theory of the $\mathrm{C}^*-al\,gebra\mathrm{C}^*(G)$ the - group is infinite cyclic, generated by the shift on $\oplus_{\mathbb{Z}}F$; the K_0- groupK_1 is free abelian of countable rank, with an explicit basis. Using the fact that G admits a 2-dimensional classifying space for proper actions, we give a direct proof of the Baum-Connes conjecture for G (which is known to hold for amenable groups, by Higson-Kasparov). As an application, we give new proofs for some results regarding full shifts in topological dynamics. This is joint work with Ramon Flores and Sanaz Pooya.

Vdovina, Alina
Ramanujan cubical complexes and non-residually finite CAT(0) groups in any dimension
We will give an explicit construction of groups acting on cube complexes of higher dimension using number theory. We will explain two consequences of our construction, namely higher dimensional Ramanujan complexes and non-residually finite groups acting on complexes of higher dimension.

Yang, Dilian
The interplay between k-graphs and the Yang-Baxter equation
I will show how k-graphs and the Yang-Baxter equation interact with each other. In particular, I will show how to apply k-graph techniques to construct an infinite family of large solutions of the Yang-Baxter equation from an arbitrarily given one; and how a single-vertex k-graph, as a semigroup, arises from a YB-semigroup.